## NORMALIZATION FORMULA FOR EXAMINATIONS CONDUCTED IN TWO OR MORE THAN TWO SHIFTS:

### Scaling Method based on cdf:-

The cumulative distribution function (cdf) describes probabilities for a random variable to fall in the intervals of the form $(-\infty, z]$. The cdf of the standard normal distribution is denoted with the capital Greek letter Φ (phi), and can be computed as an integral of the probability density function of standard normal variate from $-\infty$ to z.

$\Phi(z) = \int \varphi(z)\, dz$ where the range of integration is from $-\infty$ to z.

here $\varphi(z)$ is the standard normal pdf, $\varphi(z) = [1/\{\sqrt{(2\pi)}\}]\exp[-z^2/2]$ for $-\infty < z < \infty$.

## Numerical approximations for the normal CDF

The standard normal cdf is widely used in scientific and statistical computing. The values $\Phi(z)$ may be approximated very accurately by a variety of methods, such as numerical integration, Taylor series, asymptotic series and continued fractions. Different approximations are used depending on the desired level of accuracy.

Abramowitz & Stegun (1964) give the approximation for $\Phi(z)$ for $z > 0$ with the absolute error $|\varepsilon(z)| < 7.5\cdot 10^{-8}$

$\Phi(z) = 1 - \varphi(z)[\, b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5] + \varepsilon(z)$ where $t = 1/[1 + b_0 z]$ for $z>0$

For $z<0$ calculate the cdf as if $z>0$ (taking the magnitude of z only) by above mentioned formula and then subtract the value from 1. If $z = 0$, $\Phi(z) = 0.5$

here $\varphi(z)$ as before is the standard normal pdf, $\varphi(z) = [1/\{\sqrt{(2\pi)}\}]\exp[-z^2/2]$

and $b_0 = 0.2316419$, $b_1 = 0.319381530$,

$b_2 = -0.356563782$, $b_3 = 1.781477937$, $b_4 = -1.821255978$, $b_5 = 1.330274429$

We now describe our scaling method that uses cdf.

Let z be the z score of a candidate

That is, z= (candidate's score – mean score)/std. deviation for a particular optional subject.

Obtain Φ(z) as above which converts the z score into a probability scale [0, 1], where a candidate with average performance has z score 0 and consequently Φ(z) = 0.5 and this is true for any optional subject irrespective of the trait..

If the full marks is M we multiply Φ(z) by M to get the scaled score of the candidate in the range [0, M]. For example, if M= 100, a candidate with average performance gets z=0, $\Phi(z) = 0.5$ and $0.5 \times 100 = 50$ as his/her scaled score. In other words, simply by looking at the scaled score of a candidate in a particular optional subject whether below M/2, equal to M/2 or above M/2 one can say that the candidate has performed below average, as good as average or above average in this optional subject which is very desirable.

**Steps for calculating the scaled marks of a candidate:-**

Consider several batches for a particular subject say batch I, batch II etc.
Let us suppose there are n students in batch I and their marks are respectively $x_1, x_2, x_3, \ldots x_n$.

Step 1: Calculate the batch mean = $(x_1 + x_2 + x_3 + \ldots + x_n)/n$
Step 2: Calculate the batch standard deviation =
$\sqrt{[\{(x_1 - \text{batch mean})^2 + (x_2 - \text{batch mean})^2 + \ldots + (x_n - \text{batch mean})^2\}/n]}$
Remark: The expression inside [ ] is the batch variance. Standard deviation is the positive square root of variance.
Step 3: Calculate z score = (candidate's marks – batch mean ) / batch standard deviation
Step 4: Calculate $\varphi(z) = 1 - \{1/\sqrt{(2\pi)}\}\exp\{-z^2/2\}[b_1t + b_2t^2 + b_3t^3 + b_4t^4 + b_5t^5]$ for $z>0$
For $z<0$ first calculate $\varphi(z)$ by the formula given above taking the absolute value(i.e. magnitude only ignoring sign) of z and then subtract the value of $\varphi(z)$ from 1.

Here $t = 1/[1 + b_0z]$ and $b_0 = 0.2316419$ $b_1 = 0.319381530$
$b_2 = -0.356563782$ $b_3 = 1.781477937$ $b_4 = -1.821255978$ $b_5 = 1.330274429$
Step 5: Calculate the scaled score of the candidate = M x $\varphi(z)$ where M = Full marks.

Example:

Let full marks = 200 for a certain subject in which there are two batches of 5 students each.

| Candidate | Batch I Marks obtained | Candidate | Batch II Marks obtained |
|---|---|---|---|
| A1 | 123 | B1 | 118 |
| A2 | 158 | B2 | 107 |
| A3 | 186 | B3 | 93 |
| A4 | 92 | B4 | 100 |
| A5 | 105 | B5 | 82 |

Consider batch I
Batch mean = 132.8
Batch standard deviation = 34.63755
For the candidate A1, z score = (123- 132.8)/34.63755 = -0.28293
As $z<0$ so we take its absolute value 0.28293
With this value of z we calculate $\varphi(z)$ using the formula in step 4 and subtract the value of $\varphi(z)$ so obtained from 1 to get the correct $\varphi(z) = 0.3886153$
Since full marks = 200, Scaled score of A1 = 200 x 0.3886153 = 77.72306

Similarly for candidate A3, z score = (186-132.8)/34.63755 = 1.535905
As $z>0$ we simply calculate $\varphi(z) = 0.937719$
Scaled score of A3 = 200 x 0.937719 = 187.5438

Now consider batch II.
Batch mean = 100
Batch standard deviation = 12.21475

For candidate B4, z score = (100-100)/12.21475 = 0
$\varphi(z) = 0.5$
Scaled score of B4 = 0.5 x 200 = 100
For candidate B1, z score = (118-100)/12.21475 = 1.473628
$\varphi(z) = 0.929709$
Scaled score of B1 = 200 x 0.929709 =185.9418
Similarly for candidate B3, z score = (93-100)/12.21475 = -0.5730775
Since $z<0$ we first calculate $\varphi(z)$ taking the magnitude of z = 0.5730775 and then subtract the value of $\varphi(z)$ so obtained from 1 to get the correct $\varphi(z) = 0.283296$
Scaled score of B3 = 200 x 0.283296=56.6592